Heat Transfer from Extended Surfaces

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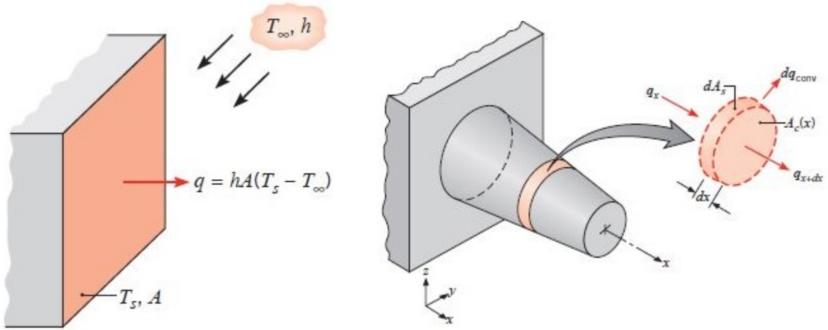
Why Extended Surface?



Analysis of heat transfer from extended surface

Assumptions:

- One dimensional;
- Steady state;
- Constant thermal; conductivity, k = constant;
- Radiation from surface is negligible;
- No internal heat generation;
- h is uniform over the surface;

Energy balance at the differential element:

$$\begin{pmatrix} \text{Rate of Heat} \\ \text{Conduction into} \\ \text{the element at x} \end{pmatrix} = \begin{pmatrix} \text{Rate of heat} \\ \text{conduction from} \\ \text{the element at x+dx} \end{pmatrix} + \begin{pmatrix} \text{Rate of heat} \\ \text{convection} \\ \text{from the element} \end{pmatrix}$$

$$\Rightarrow q_x = q_{x+dx} + dq_{conv} \tag{1}$$

Analysis of heat transfer from extended surface

From Fourier's law,

$$Heat\ conduction\ rate\ at\ x,\ q_x = -KA_c\frac{dT}{dx} \tag{2}$$

Where, A_c= Cross Sectional Area, = f(x)

$$Heat\ conduction\ rate\ at\ x+dx,\ q_{x+dx} = q_x + \frac{dq_x}{dx}dx \tag{3}$$

(Truncated taylor series expansion)

$$\Rightarrow q_{x+dx} = -KA_c\frac{dT}{dx} - K\frac{d}{dx}(A_c\frac{dT}{dx})dx \tag{4}$$

$$Convection\ heat\ transfer\ rate,\ dq_{conv} = hdA_s(T - T_\infty) \tag{5}$$

Where, A_s = Surface area of the element.

Analysis of heat transfer from extended surface

$$\Rightarrow \frac{d}{dx}(A_c \frac{dT}{dx}) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0 \tag{6}$$

$$\Rightarrow \frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\frac{dT}{dx} - (\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx})(T - T_\infty) = 0 \tag{7}$$

Let's assume the cross sectional area is uniform, $A_c \neq f(x)$

$$A_s = P \ \ x, P = fin \ \ perimeter$$

$$\Rightarrow \qquad \mathrm{dA}_s = Pdx$$

$$\frac{d^2T}{dx^2} - \frac{h}{k}\frac{P}{A_c}(T - T_\infty) = 0 \tag{8}$$

Further simplify this equation by defining an excess temperature,

$$\theta(x) = T(x) - T_\infty \tag{9}$$

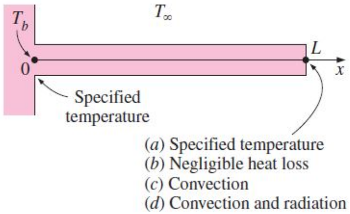
Analysis of heat transfer from extended surface

$$Let's\ assume,\ m^2 = \frac{h}{k}\frac{P}{A_c} \tag{10}$$

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{11}$$

It's a linear, homogeneous, second-order differential equation with constant coefficients. Its general solution is of the form,

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{12}$$



Thank You